Taxes, traffic jam and spillover in the metropolis

Online Appendix

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Abstract

This online appendix is organized as follows. Section 1 presents and interprets the rules characterizing the Pareto efficient allocation of resources and the efficiency-supporting local public policies. Section 2 proves the results of section 1. Section 3 proves Corollaries 1 and 2. Section 4 provides illustrations of the resource misallocations entailed by the second-best distortions in conditions (34a), (34b), and (37)–(39).

1. Pareto efficiency: Results

In this section we characterize Pareto efficiency in the MA. To this aim, we characterize the Pareto efficient allocation of resources in the MA (subsection 1.1) and determine the setting of the local policy instruments which allows to sustain this Pareto efficient allocation (subsection 1.2).

1.1. Efficient allocation

To characterize the efficient allocation, consider the problem of a benevolent central planner ignoring the private behaviors described in section 3 of the main paper and choosing directly the allocation of private and public goods, capital, residents and workers to each municipality. As standard in models with residential mobility (e.g. Wellisch, 2006), we assume that the central planner accounts for household mobility which rules out inter-jurisdiction utility differential. We can therefore assume that the central planner maximizes the utility of a resident

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of the city for instance. Formally, the program of the central planner is to maximize:

$$x_{cc} + U^c + U^{cc}$$

choosing x_{sc}, x_{jj}, G_j, K_j, R_j and W_j with $j \in \{c; s\}$, subject to (e.1)–(e.3) and:[61]

$$x_{ss} + U^s + U^{ss} = x_{cc} + U^c + U^{cc} \tag{OA.1}$$

$$x_{sc} + U^s + U^{sc} = x_{cc} + U^c + U^{cc} \tag{OA.2}$$

$$F^c + nF^s = R_c x_{cc} + n(W_{sc}x_{sc} + W_s x_{ss}) + C^c + nC^s + TC \tag{OA.3}$$

where:

$$TC \equiv n\int_{-\frac{R_c}{n}}^{R_s - W_s} T(l)\mathrm{d}l + 2n\int_{R_s-W_s}^{R_s-\frac{W_s}{2}} T(l)\mathrm{d}l \tag{OA.4}$$

is the aggregated commuting cost in the MA. Conditions (e.1), (e.2) and (e.3) are the capital, population and labor resource constraints in the MA. Conditions (OA.1) and (OA.2) are the migration conditions which state that residents mobility equalizes utility throughout the MA for each type of residents: the residents-workers of the city (cc), those of the towns (ss) and the commuters from the towns to the city (sc). Condition (OA.3) is the overall resource constraint of the MA. It states that the production in all jurisdiction, $F^c + nF^s$ covers the cost of the total consumption of private good $R_c x_{cc} + n(W_{sc}x_{sc} + W_s x_{ss})$ and that of local public good $C^c + nC^s$, as well as the total commuting cost TC. Solving the above problem, it can be shown (see the next section) that the efficient allocation of workers W_j, residents R_j, capital, K_j, local public goods G_j private goods x_{sc} and x_{jj}, across municipalities $j \in \{c; s\}$ in the MA is characterized by:

$$F^c_W - x_{sc} - t\frac{W_c}{n} - a\left(\frac{W_c}{n}\right)^2 = F^s_W - x_{ss} - t\frac{W_s}{2} - a\left(\frac{W_s}{2}\right)^2, \tag{OA.5}$$

$$x_{cc} + F^c_L = x_{\mathrm{sc}} + F^s_L, \tag{OA.6}$$

$$F^c_K = F^s_K, \tag{OA.7}$$

$$R_c(U^c_G + U^{cc}_G) + nW_{\mathrm{sc}}U^{sc}_G = C^c_G, \tag{OA.8}$$

$$R_s U^s_G + W_s U^{ss}_G = C^s_G, \tag{OA.9}$$

and the constraints (e.2)–(e.1) and (OA.1)–(OA.3).

⋆ *Labor allocation.* Condition (OA.5) characterizes the efficient allocation of workers in the MA, which requires to equalize net marginal benefits of hosting a new worker among

[61] Notice that the central planner does not distinguish towns since they are identical.

municipalities. Given residential population in each municipality, determining the efficient allocation of workers simply consists in choosing whether a suburbanite should commute to the city — left-hand side (LHS) of (OA.5) — or work in her home town — right-hand side (RHS) of (OA.5).[62] A new worker in municipality j induces an additional production of F_W^j, but entails two costs: (1) she consumes private goods (x_{sc} or x_{ss}); (2) to go to work, she pays commuting cost composed of a distance cost — tW_c/n or $tW_s/2$ — and a traffic congestion cost — $a(W_c/n)^2$ or $a(W_s/2)^2$.[63]

⋆ *Population allocation.* Condition (OA.6) characterizes the efficient allocation of residents in the MA, which requires to equalize marginal costs of residents between the city and the towns.[64] An additional resident is not only costly because she consumes private good (x_{cc} or x_{sc}) as already seen, but also because she crowds out one unit of business land which reduces the output produced in j by F_L^j.[65]

⋆ *Capital allocation.* Condition (OA.7) characterizes the efficient allocation of capital in the MA, which requires to equalize the marginal gains of allocating capital to each municipality j, i.e. the marginal output F_K^j.

⋆ *Public good provision.* Conditions (OA.8) and (OA.9) characterize the efficient provision of local public good in, respectively, the city and the towns. These are the modified Samuelson rules described in subsection 4.1 of the main paper.

1.2. Efficient local public policies

Now, suppose that the central planner does not directly chooses the allocation of workers, residents and capital in the MA. Instead, it chooses local taxes on labor τ_j^W, residents τ_j^R and capital τ_j^K to sustain efficiency, given firms and household decentralized decision making. The following result can be derived:

Result. *Necessary and sufficient conditions for local public goods, workers, residents and capital to be efficiently allocated in the MA are that the central planner chooses local public*

[62] Graphically, this it consists in choosing the location of $\mathcal{B}$ between 0 and $\mathcal{C}$ in Figure 1.

[63] Indeed, the marginal (suburbanite) worker located at $\mathcal{B}$ has to travel W_c/n distance units if commuting to the city, and $W_s/2$ distance unit if working at home.

[64] Condition (OA.6) depicts a trade-off between living in the city and commuting from the suburb to the city. But commuters can easily be replaced by residents-workers of the suburb since $x_{ss} = x_{sc} + U^{sc} - U^{ss}$ from (OA.1) and (OA.2).

[65] Recall that the total land endowment of a municipality is fixed. Then, when choosing its policy, the town arbitrates between hosting residents and hosting businesses. This mechanism is similar to that identified in Ly (2018a).

good provision in accordance with (OA.8) *and* (OA.9)*, while taxes on labor capital and residents verify:*

$$\tau_c^W - \tau_s^W = \Phi \tag{OA.10}$$

$$\tau_c^R - \tau_s^R = t\left(\frac{W_c}{n} - \frac{W_s}{2}\right) + \Phi \tag{OA.11}$$

$$\tau_c^K - \tau_s^K = 0 \tag{OA.12}$$

and the land taxes $\tau_j^{\mathcal{L}}$*,* $j \in \{c; s\}$ *allow to clear the budget constraints* (20)*.*

Proof. See the next section. □

Conditions (OA.8)–(OA.12) and (20) represent the way local governments should set their policy instruments $\{G_j; \tau_j^W; \tau_j^R; \tau_j^K; \tau_j^{\mathcal{L}}\}$ in a decentralized equilibrium so as to ensure efficiency. They are a baseline to evaluate decentralized equilibria in the remainder of the paper. Specifically, conditions (OA.10)–(OA.12) characterize an efficiency enhancing tax setting.[66]

⋆ *Labor taxation.* Condition (OA.10) states that workers are allocated efficiently in the MA if the city sets a higher labor tax than the towns. Since the city concentrates an significant amount of the workforce of the MA (Assumption 2), traffic congestion in the city is higher than in the suburb, as represented by $\Phi > 0$. The city must internalize this cost disadvantage by rising its labor tax compared to towns.

⋆ *Residential taxation.* Condition (OA.11) indicates that the same is true for residential taxes. However, it shows that, contrary to labor taxes, residential taxes in the city should be higher even in the absence of traffic congestion ($a = 0$). Condition (OA.11) shows that whenever commuting is costly ($t > 0$), the city shall rise its residential tax to above that of the towns. The reason is that concentration of labor in the city imply that commuting to the CBD is more costly than commuting to the SBD, since the marginal worker whose indifferent between both alternatives is further from the CBD.

⋆ *Capital taxation.* Condition (OA.12) indicates that city and towns should set the same capital tax rates, since capital exerts no externality in the present framework.

2. Pareto efficiency: Proofs

The purpose of this section is first to derive the results stated in the appendix section Pareto efficiency: Proofs. Specifically, we show that the central planner choices leads to conditions

[66] As showed in the next section, conditions (OA.10)–(OA.12) are strictly equivalent to conditions (OA.5)–(OA.7), when private agents behavior is accounted for.

(OA.5)–(OA.12). Second, this appendix proves Corollary 1 and Corollary 2.

2.1. *Efficient allocation*

Let us first prove the conditions characterizing the efficient allocation (OA.5)–(OA.9). The problem of the central planner is to maximize:

$$x_{cc} + U^c + U^{cc}$$

choosing x_{sc}, x_{jj}, G_j, K_j, R_j and W_j with $j \in \{c; s\}$, subject to:

$$x_{\text{ss}} + U^s + U^{s,s} - (x_{cc} + U^c + U^{c,c}) = 0 \qquad (\gamma_1)$$

$$x_{\text{sc}} + U^s + U^{s,c} - (x_{cc} + U^c + U^{c,c}) = 0 \qquad (\gamma_2)$$

$$F^c + nF^s - [R_c x_{cc} + n(R_s - W_s)x_{sc} + nW_s x_{ss} + TC + C^c + nC^s] = 0 \qquad (\gamma_3)$$

$$\mathcal{P} - W_c - nW_s = 0 \qquad (\gamma_4)$$

$$\mathcal{P} - R_c - nR_s = 0 \qquad (\gamma_5)$$

$$\mathcal{K} - K_c - nK_s = 0 \qquad (\gamma_6)$$

where γ_i, $i \in [\![1, 6]\!]$ are the Lagrange multipliers, and:

$$\begin{aligned} TC &= n\int_{-\mathcal{A}}^{\mathcal{B}}\int_{-\mathcal{A}}^{l}[t + a(\mathcal{B} - z)]\mathrm{d}z\mathrm{d}l + 2n\int_{\mathcal{B}}^{\mathcal{C}}\int_{l}^{\mathcal{C}}[t + a(z - \mathcal{B})]\mathrm{d}z\mathrm{d}l \\ &= \frac{1}{6}n(\mathcal{A} + \mathcal{B})^2(2a(\mathcal{A} + \mathcal{B}) + 3t) - \frac{1}{3}n(\mathcal{B} - \mathcal{C})^2(2a(\mathcal{B} - \mathcal{C}) - 3t) \end{aligned} \qquad \text{(OA.13)}$$

is the aggregate commuting cost defined in (OA.4). The first-order conditions with respect to x_{cc}, x_{ss} and x_{sc} are respectively:

$$1 - \gamma_1 - \gamma_2 - R_c\gamma_3 = 0 \qquad \text{(OA.14)}$$

$$\gamma_1 - nW_s\gamma_3 = 0 \qquad \text{(OA.15)}$$

$$\gamma_2 - n\left(R_s - W_s\right)\gamma_3 = 0* \qquad \text{(OA.16)}$$

The first-order conditions with respect to G_c and G_s are respectively:

$$-\gamma_3 C_G^c + \left(1 - \gamma_1 - \gamma_2\right)U_G^c + U_G^{c,c} - \gamma_1 U_G^{c,c} - \gamma_2 U_G^{c,c} + \gamma_2 U_G^{s,c} = 0 \qquad \text{(OA.17)}$$

$$-n\gamma_3 C_G^s + \left(\gamma_1 + \gamma_2\right)U_G^s + \gamma_1 U_G^{s,s} = 0 \qquad \text{(OA.18)}$$

The first-order conditions with respect to R_c and R_s are respectively:

$$-\gamma_5 - \gamma_3\left(\frac{\left(R_c + n\left(R_s - W_s\right)\right)\left(aR_c + n\left(aR_s - aW_s + t\right)\right)}{n^2} + x_{cc} + F_L^c\right) = 0 \qquad \text{(OA.19)}$$

$$-n\gamma_5 - n\gamma_3 \left(\frac{\left(R_c + n\left(R_s - W_s\right)\right)\left(aR_c + n\left(aR_s - aW_s + t\right)\right)}{n^2} + x_{\text{sc}} + F_L^s \right) = 0 \tag{OA.20}$$

The first-order conditions with respect to K_c and K_s are respectively:

$$-\gamma_6 + \gamma_3 F_K^c = 0 \tag{OA.21}$$

$$-n\gamma_6 + n\gamma_3 F_K^s = 0 \tag{OA.22}$$

The first-order conditions with respect to W_c and W_s are respectively:

$$-\gamma_4 + \gamma_3 F_W^c = 0 \tag{OA.23}$$

$$-n\gamma_4 + n\gamma_3 \left(\frac{\left(2(R_c + nR_s) - 3nW_s\right)\left(2a(R_c + nR_s) - anW_s + 2nt\right)}{4n^2} + x_{\text{sc}} - x_{\text{ss}} + F_W^s \right) = 0 \tag{OA.24}$$

Inserting (OA.14)–(OA.16) into (OA.17) and (OA.18), proves conditions (OA.8) and (OA.9). Adding (OA.14), (OA.15) and (OA.16) yields $\mu_3 = 1/(R_c + nR_s) \neq 0$. Then, multiplying (OA.19) by n and subtracting (OA.20) proves condition (OA.6). Similarly, multiplying (OA.21) by n and subtracting (OA.22) proves condition (OA.7). From the fourth and fifth constraints of the planner's problem, we have:

$$R_c + nR_s = W_c + nW_s \tag{OA.25}$$

Multiplying (OA.23) by n, subtracting (OA.24), eliminating $R_c + nR_s$ using (OA.25) and collecting terms leads to:

$$F_W^c - F_W^s + x_{\text{ss}} - x_{\text{sc}} = a\left(\frac{W_c^2}{n^2} - \frac{W_s^2}{4}\right) + t\left(\frac{W_c}{n} - \frac{W_s}{2}\right)$$

which proves (OA.5).

2.2. *Efficiency supporting taxation*

Let us now prove that, accounting for private behavior, the efficiency conditions (OA.5), (OA.6) and (OA.7) proved above are equivalent to the taxation rules (OA.10), (OA.11) and (OA.12). To this aim, we assume that the behavior described in section 3 of the main paper hold.

First, let us prove that the Pareto-efficient condition (OA.7) is equivalent to the taxation rule (OA.12) in a private economy. Optimal demand for capital and capital mobility entail condition (e.4) :

$$F_K^c - \tau_c^K = F_K^{s_i} - \tau_{s_i}^K \tag{OA.26}$$

this condition is always satisfied in the private economy. Then, a necessary and sufficient condition for the Pareto-efficient condition (OA.7) to be satisfied is:

$$\tau_c^K = \tau_s^K$$

which is condition (OA.12).

Second, let us prove that the Pareto-efficient condition (OA.6) is equivalent to to the taxation rule (OA.11) in a private economy. The budget constraints of the residents at location $-\mathcal{A}$ and B are:

$$x_{cc} + \rho(-\mathcal{A}) = w_c + rk + \Gamma - \tau_c^R \tag{OA.27}$$

$$x_{sc} + \rho(\mathcal{B}) = w_c - \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + kr + \Gamma - \tau_s^R \tag{OA.28}$$

Subtracting (OA.27) and (OA.28) yields:

$$x_{cc} + \rho(-\mathcal{A}) + \tau_c^R = x_{sc} + \rho(\mathcal{B}) + \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + \tau_s^R$$

substituting $\rho(-\mathcal{A})$ and $\rho(\mathcal{B})$ using (A.5) and (18), we obtain:

$$x_{cc} + F_L^c + \tau_c^R = x_{sc} + w_c - \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + \Lambda^{sc} + \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + \tau_s^R$$

Eliminating w_c and Λ^{ss} using respectively (e.6) and (19) implies:

$$x_{cc} + F_L^c + \tau_c^R = x_{sc} + F_L^s - \int_{\mathcal{B}}^{\mathcal{C}} [t + a(z - B)]\mathrm{d}z + \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + \tau_s^R$$

Inserting the definitions of A, B and C, (3)–(5), and integrating entails:

$$x_{cc} + F_L^c + \tau_c^R = x_{sc} + F_L^s + \frac{(2(R_c + nR_s) - 3nW_s)\,(2a(R_c + nR_s) - anW_s + 4nt)}{8n^2} + \tau_s^R$$

Eliminating $R_c + nR_s$ using (OA.25) and collecting terms leads to:

$$x_{cc} + F_L^c + \tau_c^R = x_{sc} + F_L^s + t\left(\frac{W_c}{n} - \frac{W_s}{2}\right) + \frac{a}{2}\left(\frac{W_c^2}{n^2} - \frac{W_s^2}{4}\right) + \tau_s^R \tag{OA.29}$$

which is always satisfied in the private economy. Then, a necessary and sufficient condition for the Pareto-efficient condition (OA.6) to be satisfied is:

$$x_{cc} + F_L^c + \tau_c^R = x_{sc} + F_L^s + t\left(\frac{W_c}{n} - \frac{W_s}{2}\right) + \frac{a}{2}\left(\frac{W_c^2}{n^2} - \frac{W_s^2}{4}\right) + \tau_s^R$$

which is condition (OA.11).

Finally, let us prove that the Pareto-efficient condition (OA.5) is equivalent to to the taxation rule (OA.10) in a private economy. The budget constraints of the residents at location B and C are:

$$x_{sc} + \rho(\mathcal{B}) = w_c - \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z + kr + \Gamma - \tau_s^R \tag{OA.30}$$

$$x_{ss} + \rho(\mathcal{C}) = w_s + rk + \Gamma - \tau_s^R \tag{OA.31}$$

Subtracting (OA.30) and (OA.31) yields:

$$x_{sc} + \rho(\mathcal{B}) - w_c + \int_{-\mathcal{A}}^{\mathcal{B}} [t + a(\mathcal{B} - z)]\mathrm{d}z = x_{ss} + \rho(\mathcal{C}) - w_s$$

substituting $\rho(\mathcal{B})$ and $\rho(\mathcal{C})$ using (A.5) and (18), and following the same computation step as above, we obtain:

$$x_{sc} - w_c + t\left(\frac{W_c}{n} - \frac{W_s}{2}\right) + \frac{a}{2}\left(\frac{W_c^2}{n^2} - \frac{W_s^2}{4}\right) = x_{ss} - w_s$$

Substituting the wages using the input demand condition (15):

$$x_{sc} - F_W^c + \tau_c^W + t\left(\frac{W_c}{n} - \frac{W_s}{2}\right) + a\left(\frac{W_c^2}{n^2} - \frac{W_s^2}{4}\right) - \Phi = x_{ss} - F_W^s + \tau_s^W \tag{OA.32}$$

where Φ is as defined in (25). This condition is always satisfied in the private economy. Then, a necessary and sufficient condition for the Pareto-efficient condition (OA.5) to be satisfied is:

$$\tau_c^W - \tau_s^W = \Phi$$

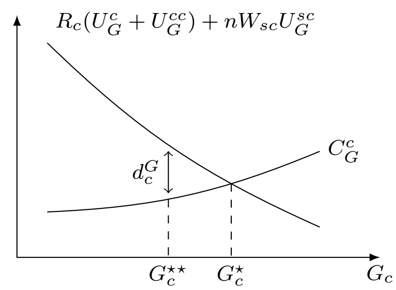
which is condition (OA.10).

3. Proofs of Corollaries 1 and 2

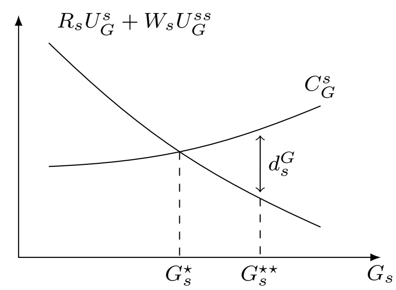
Let us prove Corollary 1, that is, conditions (26)–(31). First, notice that the taxation rules (26)–(28) directly follow from subtracting (21a) to (21b), (22a) to (22b) and (23a) to (23b). Since the taxation rules (26)–(28) are strictly identical to those of the central planners' rule (OA.10)–(OA.12), the proofs in the preceding subsection imply that conditions (29)–(31) also hold.

Let us now prove Corollary 2, that is, conditions (35)–(39). The taxation rules (35) and (36) directly follow from subtracting (32a) to (32b) and (33a) to (33b). Setting $\tau_j^W = 0$, $j \in \{c; s\}$ in condition (OA.32) — which holds at the equilibrium — proves condition (39). Finally, inserting (35) and (36) into the equilibrium conditions (OA.29) and (OA.26) proves conditions (37) and (38).

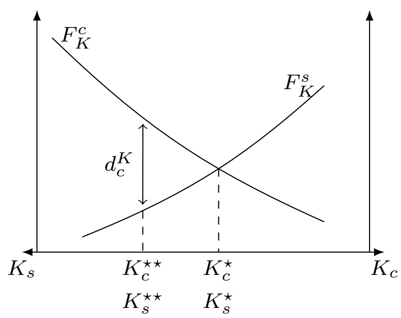
4. Illustration of the distortions



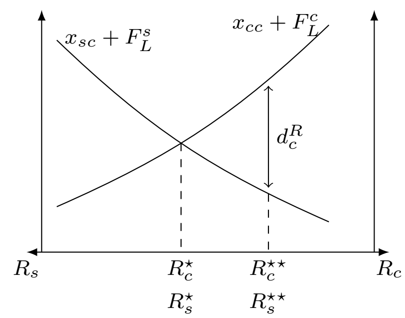
(a) Local public goods in the city.



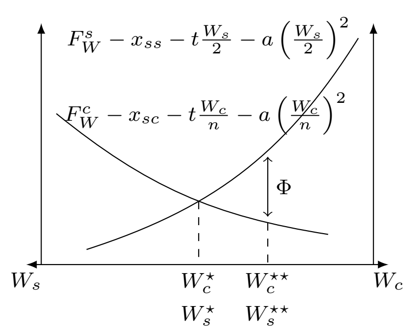
(b) Local public goods in town s.



(c) Capital in the city and town s.



(d) Residents in the city and town s.



(e) Workers in the city and town s.

Figure OA.1. Misallocation due to the absence of local labor taxes.